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COMMON STOCHASTIC FEATURES AND THEIR ECONOMIC INTERPRETATION

Abstract

Background: Cointegration analysis has been part of the research literature for almost 40 years. However, other common features that cause disequilibrium of economic categories have so far attracted less attention. This leads to an interesting question about the degree to which studies in the alternative fields are substitutive or complementary, and what conditions must be met in order to undertake the appropriate analysis (cointegration, co-cyclical, co-autocorrelation or other, less frequently discussed co-behaviors) is purposeful.
Research purpose: The purpose of this paper is the comparison of different types of common stochastic behaviors. The type of common factors and the resulting analysis of movements that should be chosen, naturally depends on the time horizon, which can be long, medium, or short, but a reliable study should not ignore any of these perspectives. This study tries to demonstrate that the key role in this choice is played by the reduced rank of the most important matrices that occur in the appropriate VAR model representations or the isomorphic representations thereof. Another research goal was to show that the above-mentioned analyses of stochastic co-movements are largely complementary.
Methods: Multidimensional dynamic econometrics based on VAR models was selected for the study because it contains tools that enable the different methods of analyzing common behaviors to be analyzed. Possible combinations of full and reduced cointegrating matrix ranks and the medium- and long-run relationships matrices were considered and economically interpreted. Relationships between the matrices have been identified, and the iterative mechanism that causes the system to return to equilibrium is described.
Conclusions: The study confirms that the analyzed investigations on common dominant components were essentially complementary. Extending the analysis to seasonal cointegration or deterministic co-trending would allow substitutive elements to be revealed. For example, a cointegration analysis using a relatively short time horizon is an alternative to co-trending (the stochastic trend expires only in a very long perspective), and an analysis that considers a more integrated process could be an alternative to co-deterministic cyclical analysis.

Keywords: cointegration, co-autocorrelation, equilibrium and adjustment mechanisms, shocks.

JEL classification: C01, C32, C52

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1. Introduction

For many years, researchers analyzing co-movements have directed their attention to cointegration. Given its relationship with long-run economic equilibrium, it is admittedly the most important but not the only example of factors inherent in the common behaviors of data generating processes (DGPs).

This work's main focus is on the co-features and common features of the stochastic part of DGPs, i.e., cointegration, stochastic co-cyclicality and co-autocorrelation. In the short-term (ST), under certain conditions, the co-MA (co-moving average) may become similar to cointegration *au rebours*. This cointegration type (considered only for the negative integration order d) is different from the classical type in that it is short-run and, primarily, as a result, the random component integration order increases (not decreases) with respect to the variables' integration order. Also, the medium-run cointegration from the I(2) analysis is more similar to stochastic co-cyclicality than to classical cointegration.

Due to editorial limitations, the research scope is limited to the time domain, so seasonal cointegration, which involves analysis in the non-zero frequency domain, is only mentioned without delving into it.

The paper is structured as follows. In section 2, co-movements are compared, while section 3 focuses on the importance of the ranks of full and reduced matrices that occur in vector autoregression (VAR) models. Section 4 explores the adjustment mechanisms present in the more complex VAR models, especially the role of stocks, flows and accelerant shocks. Section 5 concludes.

2. Cointegration and other co-movements

Table 1 shows the different types of factors inherent in DGP. For simplicity's sake, this table is incomplete. Notwithstanding, it makes it possible to examine other co-feature types not only in the long run, but also in the short and medium run.

TABLE 1: *Common stochastic factors in DGP*

Factor	Long-run	Medium-run	Short-run
1	2	3	4
Stochastic I(1) trend (I(1) domain)	CI(1,1)		
Stochastic I(2) trend	CI(2,2), CI(2,1)	CI(2,2)	

1	2	3	4
Stochastic cycle	vanished	CI(1,1) in I(2) domain	
Nonstationary stochastic cycle	cointegration in the non-zero frequency domain (wider discussion in Gregoir and Laroque)*	seasonal cointegration	
Switch in stochastic structure	co-stochastic breaking (common stochastic structural change)		
I(0) autocorrelation			co-serial correlation
MA component in DGP			co-MA
ARCH effect	co-ARCH		
Heteroscedasticity	co-heteroscedasticity		

Explanations:
* **S. Gregoir**, **G. Laroque**, *Multivariate Time Series: A Polynomial Error Correction Representation Theorem*, Econometric Theory 1993/9/3, pp. 329–342.
Source: own study.

As Table 1 shows, stochastic co-factors are accompanied by many deterministic co-factors, which will not be dealt with in this paper. Seasonal cointegration will not be considered either because of its special nature that combines cointegration and stochastic co-seasonality. Discussing it thoroughly would require extending the analysis to the frequency domain (the paper's focus is on zero frequency).

For many years, traditional VAR models and their vector error correction model VECM transformations should contain additional components. Let us consider a simplified model that contains only stochastic components:

$$\Delta \mathbf{Y}_t = \mathbf{\Pi}\mathbf{Y}_{t-1} + \sum_{s=1}^{S-1} \mathbf{\Gamma}_s \Delta \mathbf{Y}_{t-s} + \mathbf{\Sigma}_t, \qquad (1)$$

where $\mathbf{\Pi} = \sum_{s=1}^{S} \mathbf{\Pi}^{(s)} - \mathbf{I}$ and $\mathbf{\Gamma}_s = -\sum_{j=s+1}^{S} \mathbf{\Pi}^{(j)}$

wherein $\mathbf{\Pi}^{(s)}$ is the VAR model parameters:

$$\mathbf{Y}_t = \mathbf{\Pi}^{(1)}\mathbf{Y}_{t-1} + \mathbf{\Pi}^{(2)}\mathbf{Y}_{t-2} + \ldots + \mathbf{\Pi}^{(S)}\mathbf{Y}_{t-S} + \mathbf{\Sigma}_t \tag{1a}$$

The $\mathbf{\Gamma}_s$ elements measure the transitory effects of changes in lagged variable values. $\mathbf{\Pi}$ is the total impact multiplier matrix that, under joint stationarity, is non-singular; therefore, it has full column rank M (variable space dimension).

Where at least one stochastic trend is present, the $\mathbf{\Pi}$ rank is $R < M$. Hence, the following decomposition is possible:

$$\mathbf{\Pi} = \mathbf{A}\mathbf{B}^T, \tag{2}$$

where:

$\mathbf{A} = \left[\boldsymbol{\alpha}_1\boldsymbol{\alpha}_2 \ldots \boldsymbol{\alpha}_R\right]_{M*R}$ – full column rank weights (adjustment) matrix,

$\mathbf{B} = \left[\boldsymbol{\beta}_1\boldsymbol{\beta}_2 \ldots \boldsymbol{\beta}_R\right]_{M*R}$ – full column rank matrix, which consists of the parameters that define baseline (independent) cointegrating vectors.

Under a traditional approach, the $\mathbf{\Pi}$ rank is determined by the $\mathbf{B}$ rank and, thereby, by the cointegrating space dimension because the cointegrating vectors that comprise the matrix are, by definition, linearly independent. The α_{mr} elements are the weights that should be assigned to the r-th baseline cointegration relationship to explain its long-run impact on the m-th variable of the model. They do not have to be in the interval (0, 1) and do not have to add up to one.

Only when the I(1) processes are "pure" random walks whose realizations (variables) change according to Brownian motion are their increments spherical. Accelerations of cumulative random walks vary in a purely random manner. Such processes are rare in economic practice; most economic variables are generated by DGPs that are not significantly different from processes integrated of corresponding integer order. Their properties differ from the "model" processes only asymptotically. From the perspective adopted by economic entities, such a simplification is fully justified. While a difference filter can free the process from stochastic trends, cycles, or both, it does not solve the DGP's autocorrelation problem, the ARCH effect, or the moving average. The transfer of these effects from the DGP that generates the variables to random components from the relationships between these variables has serious consequences. From the statistical properties perspective, the equilibrium dependences estimator in the system is super-consistent when the variables are cointegrated,[1] and if the cointegration of variables concerns the I(2) system, it is even super-super-consistent but not efficient. This means that although the estimation precision improves quickly (the I(1) system) or even very quickly (the I(2) system), it continues to be limited for a relatively long period.

Π rank $R < M$ implies that the solution to models (1)–(4) for past and current shocks is not a relatively simple vector moving average model. It is necessary to use the Beveridge and Nelson[2] decomposition, which can separate permanent shocks (stochastic trends) from transitory shocks (cf. Engle and Granger[3]).

$$\mathbf{Y}_t = \mathbf{C}\sum_{i=1}^{t}\Sigma_i + \mathbf{C}(L)\Sigma_t, \tag{3}$$

where:

$$\mathbf{C} = \mathbf{B}_\perp(\mathbf{A}_\perp^T(\sum_{s=1}^{S-1}\mathbf{\Gamma}_s - \mathbf{I})\mathbf{B}_\perp)^{-1}\mathbf{A}_\perp^T, \tag{3a}$$

$\mathbf{A}_\perp = \left[\overline{a}_{ij}\right]$, $\mathbf{B}_\perp = \left[\overline{b}_{ij}\right] - M * (M - R)$ are orthogonal complements of a **A** and **B** respectively; the full column rank of both matrices is assumed. Derivation (3a) is given by Johansen[4] and Wróblewska.[5]

1 **J.H. Stock**, *Asymptotic Properties of Least-Squares Estimators of Co-integrating Vectors*, Econometrica 1987/55, pp. 1035–1056.

2 **S. Beveridge**, **Ch. Nelson**, *A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the ‘business cycle’*, North-Holland Publishing Company, Journal of Monetary Economics 1981/7, pp. 151–174.

3 **R.F. Engle**, **C.W.J. Granger**, *Cointegration and Error Correction: Representation, Estimation and Testing*, Econometrica 1987/55, pp. 251–276.

4 **S. Johansen**, *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford 1995.

5 **J. Wróblewska**, *Bayesian Analysis of Weak Form Polynomial Reduced Rank Structures in VEC Models*, Central European Journal of Economic Modelling and Econometrics 2012/4/4, pp. 253–267.

Matrix $\mathbf{A}_{\perp}$ contains coefficients that define $M - R$ independent common stochastic trends which throw system $\mathbf{A}_{\perp}^{T}\sum_{i=1}^{t}\Sigma_i$ off equilibrium. The system variables' vulnerability to permanent shocks is described by $\tilde{\mathbf{B}}_{\perp} = \mathbf{B}_{\perp}(\mathbf{A}_{\perp}^{T}\left(\sum_{s=1}^{S-1}\mathbf{\Gamma}_s - \mathbf{I})\mathbf{B}_{\perp}\right)^{-1}$.

When the I(2) stochastic trends are present, it is more convenient to consider a VECM:

$$\Delta^2\mathbf{Y}_t = \mathbf{\Pi}\mathbf{Y}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{Y}_{t-1} + \sum_{s=1}^{S-2}\mathbf{\Psi}_s\Delta^2\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \tag{4}$$

where $\mathbf{\Gamma} = \sum_{s=1}^{S-1}\mathbf{\Gamma}_s - \mathbf{I}$ is an $M * M$ mean lag matrix that describes medium-run relationships. In representation (4), $\mathbf{\Psi}_s = -\sum_{j=s+1}^{S-1}\mathbf{\Gamma}_j$ takes over the short-run relationships matrix role.

Due to the $(\mathbf{A}_{\perp}^{T}\left(\sum_{s=1}^{S-1}\mathbf{\Gamma}_s - \mathbf{I})\mathbf{B}_{\perp}\right)$ noninvertibility, from (3a), the common stochastic trends are represented by:

$$\mathbf{Y}_t = \mathbf{C}_1\sum_{i=1}^{t}\Sigma_i + \mathbf{C}_2\sum_{j=1}^{t}\sum_{i=1}^{j}\Sigma_i + \mathbf{C}(L)\Sigma_t, \tag{5}$$

where $\mathbf{C}_2 = \mathbf{B}_{2\perp}(\mathbf{A}_{2\perp}^{T}(\mathbf{\Gamma}\mathbf{B}(\mathbf{B}^{T}\mathbf{B})^{-1}(\mathbf{A}^{T}\mathbf{A})^{-1}\mathbf{A}^{T}\mathbf{\Gamma} - \sum_{s=1}^{S-2}\mathbf{\Psi}_s)\mathbf{B}_{2\perp}^{T})^{-1}\mathbf{A}_{2\perp}^{T}$ is an $M * M$ matrix of parameters for the I(2) stochastic trends given by $\sum_{j=1}^{t}\sum_{i=1}^{j}\Sigma_i$, and $\mathbf{A}_{2\perp}^{T}$ (which can be interpreted as the matrix that defines independent common stochastic trends I(2)) and $\mathbf{B}_{2\perp}$ are $M * P_2$-dimensional.

Vahid and Engle[6] improved the VECM analysis by decomposing the short-run relationships matrices $\mathbf{\Gamma}_s$. For this operation to be possible, the $\mathbf{\Gamma}_s$ ranks must be reduced and equal to $M - N$, which denotes the number of linearly independent short-run relationships between stationary I(0) first differences. The DGP for the differences is a stationary AR (not necessarily of order 1), while deviations from the short-run relationships are purely random. Their analysis focused on a system that contains only integrated processes of, at most, order one. The analysis of the desirability of such phenomena results is because classical random walks and near integrated or (first) difference stationary AR(p) processes of a higher order belong to the class of I(1) processes. Their increments are stationary but not always spherical.

With I(1), the decomposition was proposed by Vahid and Engle[7] or Engle and Kozicki[8]:

$$\Delta \mathbf{Y}_t = (\mathbf{W}\mathbf{S}_0^T)\mathbf{B}^T\mathbf{Y}_{t-1} + \sum_{s=1}^{S-1}(\mathbf{W}\mathbf{S}_s^T)\Delta\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \tag{6}$$

where: $\mathbf{A} = \mathbf{W}\mathbf{S}_0^T$, $\mathbf{\Gamma}_s = \mathbf{W}\mathbf{S}_s^T$, $s = 1,\ldots, S-1$,
$\mathbf{W}$ is $M * (M - N)$ weights matrix, which should be assigned to the baseline relationships that restore the random component white noise from the short-run (“whitening”) relationships to explain the short-run fluctuations of the consecutive variables in the system. It has a full $M - N$ column rank as do $\mathbf{S}_s^T$ ($s = 0,\ldots, S - 1$). From the basic rules on the matrix product rank, $N < M - R$. These weights (which measure adaptive reactions to deviations from short-run dependencies) stay invariant.
$\mathbf{S}_s^T$ ($s = 0,\ldots, S-1$) are $M * (M-N)$ matrices of baseline short-run dependencies that ensure white-noise deviations from them. $\mathbf{S}_0^T$ is also a basic short-run relationships matrix, as the adjustment reactions are also short-run.

6 **F. Vahid**, **R.F. Engle**, *Common Trends and Common Cycles*, Journal of Applied Econometrics 1993/8/4, pp. 341–360.

7 *Ibidem*.

8 **R.F. Engle**, **S. Kozicki**, *Testing for Common Features*, Journal of Business & Economic Statistics 1993/11/4, pp. 369–380.

Cubadda[9] and Wróblewska[10] correctly observed that the short-run "cyclical" component cannot be identified with business cycles in the I(2) or even the I(1) domain. The *N*-dimensional centrifugal common features matrices (a term proposed by Engle and Kozicki,[11] cf. Wróblewska[12]) related to the dual representation (the VECM solution in relation to past and current shocks, i.e., forces that disturb the system equilibrium) should be seen as clearly distinct from the *M* – *N*-dimensional co-features matrices (the term "co-features" was suggested by Engle and Kozicki[13] and Hecq, Palm, and Urbain[14]) related to centripetal (adaptive) system behaviors. *N* refers to common movements and *M* – *N* to equilibrium adjustments, in contrast to long-run analysis. Some authors, such as Engle and Kozicki, use both "co-feature" and "common feature," which is wise as their terminology concentrates on the similarities between DGP, not on the distinction between centrifugal and centripetal behaviors.

Engle and Kozicki suggest considering a reduced rank restriction for $\mathbf{\Gamma}$ in the I(1) system because with $\mathbf{\Gamma} = \sum_{s=1}^{S-1} \mathbf{\Gamma}_s - \mathbf{I}$, the $\mathbf{\Gamma}_s$ rank is smaller than its dimension ($M * M$), and so the $\mathbf{\Gamma}$ rank can be reduced. However, to fulfill a reduced rank restriction for $\mathbf{\Gamma}$, an additional condition, $\mathbf{S}_1^T \mathbf{\Gamma}_1 = \mathbf{0}$, is required. Consequently, $\mathbf{\Gamma}$, known especially from I(2) analysis, plays an important role in simple short-run analysis in the case of I(1).

With the I(2) system, $\mathbf{\Psi}_s$, which describes relationships between the second increments, should be decomposed, as the first increments are usually generated by I(1). If the matrices' ranks are reduced (but positive), then the autocorrelation that is inherent in the first increments of the I(1) does not transfer to random components from short-run dependencies. Only when these ranks are equal to zero can the absence of short-run relationships be assumed. As a result of analyzing the short-run relationships, restrictions are imposed on $\mathbf{\Gamma}_s$ and/or $\mathbf{A}$ (in the I(2) systems, on $\mathbf{\Psi}_s$ and/or $\mathbf{A}$, respectively) to obtain short-run dependencies

[9] **G. Cubadda**, *Common Serial Correlation and Common Business Cycles: A Caution Note*, Empirical Economics 1999/24/3, pp. 529–535.

[10] **J. Wróblewska**, *Analiza modelu realnego cyklu koniunkturalnego z wykorzystaniem bayesowskich modeli typu VEC*, Przegląd Statystyczny 2017/LXIV/4, pp. 357–372.

[11] **R.F. Engle**, **S. Kozicki**, *Testing for Common Features…*, pp. 369–380.

[12] **J. Wróblewska**, *Analiza modelu realnego cyklu koniunkturalnego...*, pp. 357–372.

[13] **R.F. Engle**, **S. Kozicki**, *Testing for Common Features…*, pp. 369–380.

[14] **A. Hecq**, **F. Palm**, **J.-P. Urbain**, *Common Cyclical Features Analysis in VAR Models with Cointegration*, Journal of Econometrics 2006/132 (1), pp. 117–141.

free of the random component autocorrelation. In the I(1) system, it is about the relationships between the first stocks increments, whereas in the I(2) system between the second stocks increments and most often, these are relationships between the flows (for I(1)) and flow changes (accelerants) in the I(2) domain.

Decomposition (6) for model (4) is written as:

$$\Delta^2 \mathbf{Y}_t = (\mathbf{W}\tilde{\mathbf{S}}_0^T)\mathbf{B}^T\mathbf{Y}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{Y}_{t-1} + \sum_{s=1}^{S-2}(\mathbf{W}\tilde{\mathbf{S}}_s^T)\Delta^2\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \tag{7}$$

where: $\mathbf{\Psi}_s = \mathbf{W}\tilde{\mathbf{S}}_s^T$, $s = 1,\ldots, S-2$.

Alternatively, after capturing restrictions for the medium-run relationships (which can only be deduced from the cited authors' deliberations on the I(1) system), it can be written:

$$\Delta^2 \mathbf{Y}_t = (\mathbf{W}\tilde{\mathbf{S}}_0^T)\mathbf{B}^T\mathbf{Y}_{t-1} + (\mathbf{W}\tilde{\tilde{\mathbf{S}}}^T)\Delta\mathbf{Y}_{t-1} + \sum_{s=1}^{S-2}(\mathbf{W}\tilde{\mathbf{S}}_s^T)\Delta^2\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \tag{8}$$

where: $\mathbf{\Gamma} = \mathbf{W}\tilde{\tilde{\mathbf{S}}}^T$.

The orthogonal complement properties imply that

$$\mathbf{W}_\perp^T(\mathbf{W}\tilde{\mathbf{S}}_0^T)\mathbf{B}^T\mathbf{Y}_{t-1} + \mathbf{W}_\perp^T(\mathbf{W}\tilde{\tilde{\mathbf{S}}}^T)\Delta\mathbf{Y}_{t-1} + \mathbf{W}_\perp^T\sum_{s=1}^{S-2}(\mathbf{W}\tilde{\mathbf{S}}_s^T)\Delta^2\mathbf{Y}_{t-s} = \mathbf{0}$$

This results in removing the short-run component from representation (3), as Vahid and Engle[15] showed for I(1) that $\mathbf{W}_\perp^T\mathbf{C}(L)\Sigma_t = \mathbf{0}$. Thus, the representation of common stochastic trends becomes homogeneous (it describes only long-run shocks).

The extension to the I(2) domain was performed by Paruolo.[16] It remains the case that $\mathbf{W}_\perp^T\mathbf{C}(L)\Sigma_t = \mathbf{0}$, but not necessarily $\mathbf{W}_\perp^T\mathbf{C}_1(L)\Sigma_t = \mathbf{0}$. As regards the solution for common stochastic I(2) trends, the pseudo-cyclical component (short-term) can be removed, but not the stochastic cycles I(1), because, as

[15] **F. Vahid**, **R.F. Engle**, *Common Trends and Common Cycles…*, pp. 341–360.

[16] **P. Paruolo**, *Common Trends and Cycles in I(2) VAR Systems*, Journal of Econometrics 2006/132, pp. 143–168.

Juselius[17] observed, it is not possible to decompose the $\mathbf{C}_1$ in representation 5 in an unambiguous, let alone economically interpretable way. The unfortunate term "cyclical component" is due to the fact that autocorrelation causes the common cyclical feature (there are N of them) to disappear. Representations (3) and (6) have a triple reduced-rank condition: for $\mathbf{\Pi}$, $\mathbf{\Gamma}_s$ and $\mathbf{A}$. The last reduced rank is a very large distinction from $\mathbf{A}$ full column rank equal to the cointegration rank R from traditional analysis. In representation (7), as many as five reduced rank conditions can be considered. Assuming that the system contains variables generated by I(2) processes, a reduced rank restriction ($P_1 < M - R$) exists, where P_1 denotes the I(1) stochastic cycles number) for $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$. Engle and Kozicki's results[18] for an I(1) system concerning the conditions for $\mathbf{\Gamma}$ reduced rank restriction remain valid. However, instead of $\mathbf{\Gamma}_s$, which does not explicitly appear in formula (4), a $\mathbf{\Psi}_s$ reduced rank analysis is advisable. For simplicity, it is assumed that all $\mathbf{\Gamma}_s$ and $\mathbf{\Psi}_s$ matrices have rank $M - N$, which is higher than R.

From the identifiability viewpoint, in both the I(1) and I(2) domains, the long-run relationships are contained in $\mathbf{B}$. The fact that the $\mathbf{A}$ rank is higher than R is not a problem. Because of the crucial features of the matrix product rank, the $\mathbf{\Pi}$ rank is still R.

The possibility of decomposing the adjustment matrix related to the reduced column rank $\mathbf{A} = \mathbf{W}\mathbf{S}_0^T$ has interesting economic interpretations. As $\mathbf{\Gamma}_s = \mathbf{W}\mathbf{S}_s^T$, in the case of a triple (and in the I(2) system, up to five times) reduced rank, two types of short-run behavior in the system (adaptive and strictly short-run) are not independent. Let us consider the α_{mr} element of $\mathbf{A}$ (an error correction matrix):

$$\alpha_{mr} = w_{m1}s_{1r} + w_{m2}s_{2r} + \ldots + w_{m,M-N}s_{M-N,r} \tag{9}$$

It is a short-run baseline "whitening" reactions function, meaning that adjusting the m-th system's variable to deviations from the long-run equilibrium depends on the adjustments to short-run reactions. Therefore, formula (9) can be referred to as a sustained (or persistent) error correction model. Model (7) contrasts with the slightly mechanical representation (4) in that it allows not only the adjustment reaction itself to be analyzed but also the distribution of short-run

[17] **K. Juselius**, *Cointegrated VAR Model. Methodology and Applications*, Oxford University Press, Oxford 2006.

[18] **R.F. Engle**, **S. Kozicki**, *Testing for Common Features*…, pp. 369–380.

reaction times that necessitate long-run adjustments. A real-life example of this can be a less experienced investor's hysterical reaction in the stock market or an insufficient or exaggerated reaction of the marginal propensity to consume (MPC) to changes in inflation. The adjustment reactions described by **A** with type (9) restrictions show the entire adjustment mechanism, including the factors that trigger it.

A very similar mechanism may be connected with the adjustment response to medium-run (co-cyclical) cointegration.

The stochastic processes integrated of negative order interpretation is rarely discussed in the literature. It can be assumed that the negative integration order problem only concerns flows, accelerants in particular. The stocks I(0) (hence flows I(–1)) case was discussed by Majsterek[19] in terms of public debt exceeding the cap set by the legislature. Interpretation is easier when the deficit is I(0), and its increment is I(–1). According to Table 2, this is the $\mathbf{\Pi}$ full rank case. An I(0), the deficit cannot cointegrate with other variables. However, controlling the short-run behavior of this fiscal policy key category is still possible, although only when residuals from the short-run relationships that explain deficit changes are stationary. Such a situation is possible only if the $\mathbf{\Gamma}_s$ rank is reduced, but not zero. This means that there are noninvertible MA(1) annihilation (co-moving average) processes, which can be described as cointegration *au rebours*. Paruolo[20] does not see the co-moving average as a separate type of co-feature and generalizes the cointegration concept for the negative *d* case. The co-MA occurrence involves not a reduction but an increase in the integration order of the process that generates residuals from the relationship between the variables. The crucial difference between the interdependence of noninvertible MA and classical cointegration is that the former concerns a short period. In contrast, classical cointegration is a long-run equilibrium relationship, in special cases only occurring in the medium-run (direct CI(2,2) or medium-run CI(1,1) in I(2) systems).

Let us consider noninvertible MA(1), such as a budget deficit increase. The $\mathbf{\Gamma}_s$ ranks equalling zero may be interpreted as the tax authorities having control over the deficit level in the long run (undesirable shocks that increase the deficit are stationary, and so they are ineffective due to their "short memory"). However, the desirable shocks that reduce the deficit are also inefficient (according to

[19] **M. Majsterek**, *Zasoby i strumienie w kontekście analizy kointegracyjnej*, Studia Prawno-Ekonomiczne 2020/CXIV, pp. 273–293.

[20] **P. Paruolo**, *Common trends and cycles in I(2) VAR systems*…, pp. 143–168.

Hamilton,[21] they increase the next period's deficit). With reduced but non-zero $\mathbf{\Gamma}_s$ ranks, the deficit is still resistant to long-run shocks. However, it can be controlled, at least in the short run, using economic policy instruments.

Let us note that the short-run interrelationships of MA processes do not need to be related to classical cointegration if they concern invertible MA (which are, by definition, always stationary). In such a case, only the "whitening" of the residual process in short-run relationships occurs. The process invertibility (consider the simplest MA(1), which means that a given variable is influenced by a current and past shock combination, the latter being less forceful). The current shock can be represented as an infinite-degree MA with respect to the current and past values of economic variables on which affects or affected:

$$\zeta_t = y_t - \kappa y_{t-1} + \kappa^2 y_{t-2} - \kappa^3 y_{t-3} + \ldots = \sum_{i=0}^{\infty} (-\kappa^i) y_{t-i} \qquad (10)$$

where κ is the standard MA(1) parameter $y_t = \zeta_t + \kappa\zeta_{t-1}$.

Formula (10) can be interpreted in economic terms. If an economic entity plans to send a shock to a given economic variable for benevolent or malevolent reasons (e.g., a speculative attack on a currency), it seems logical to design such shock, taking into account the size of the target variable. Also, a negative dependence on the previous size of this variable is a common smoothing mechanism. The MA non-invertibility that generates variables in the system prevents decision-makers from finding an algorithm to generate optimal shocks (if they intend to cause them) or predict shocks to properly prepare the economic system for further shocks. Naturally, all these deliberations are irrelevant with purely random shocks, or shocks generated by a process with a long memory (for a wider discussion, see Majsterek[22]).

All foregoing deliberations were guided by the implicit assumption on a strong form common factor (SFCF). Hecq, Palm and Urbain[23] relaxed Vahid and Engle's[24] assumptions and proposed a weak-form reduced-rank structure

[21] **J.D. Hamilton**, *Time Series Analysis*, Princeton University Press, Princeton 1994.

[22] **M. Majsterek**, *Cointegration Analysis in the Case of I(2) – General Overview*, Central European Journal of Economic Modelling and Econometrics 2012/4/4, pp. 215–252.

[23] **A. Hecq**, **F. Palm**, **J.-P. Urbain**, *Separation, Weak Exogeneity, and P–T Decomposition in Cointegrated VAR Systems with Common Features*, Econometric Reviews 2002/21 (3), pp. 273–307.

[24] **F. Vahid**, **R.F. Engle**, *Common Trends and Common Cycles*…, pp. 341–360.

(also referred to as a weak-form common factor (WFCF)). Their analysis only included double (I(1) system) or triple (I(2) system) reduced $\mathbf{\Pi}$ and $\mathbf{\Gamma}_s$ ranks for the I(1) domain, and $\mathbf{\Pi}$, $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ and $\mathbf{\Psi}_s$ for the I(2) domain. Thus, the adjustment responses to the long- and medium-run relationships were treated as independent again. Under the WFCF, $N > M - R$ is possible. Formula 5 should be replaced by

$$\Delta \mathbf{Y}_t = \mathbf{A}\mathbf{B}^T\mathbf{Y}_{t-1} + \sum_{s=1}^{S-1}(\mathbf{W}\mathbf{S}_s^T)\Delta\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \qquad (11)$$

where: $\mathbf{\Gamma}_s = \mathbf{W}\mathbf{S}_s^T$, $s = 1,\dots, S-1$.

The consequence is again the **A** full column rank, i.e., R, so the additional identifiability problem does not occur. However, this WFCF, unlike the SFCF, does not allow us to track the long-run adjustment forces mechanism establishing.

For I(2):

$$\Delta^2\mathbf{Y}_t = \mathbf{A}\mathbf{B}^T\mathbf{Y}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{Y}_{t-1} + \sum_{s=1}^{S-2}(\mathbf{W}\tilde{\mathbf{S}}_s^T)\Delta^2\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \qquad (12)$$

where: $\mathbf{\Psi}_s = \mathbf{W}\tilde{\mathbf{S}}_s^T$, $s = 1,\dots, S-2$.

Representation (10) means that, again, the long- and medium-run adjustments are independent of the short-run ones.

There may be an intermediate form between SFCF and WFSF:

$$\Delta^2\mathbf{Y}_t = \mathbf{A}\mathbf{B}^T\mathbf{Y}_{t-1} + (\mathbf{W}\tilde{\tilde{\mathbf{S}}}^T)\Delta\mathbf{Y}_{t-1} + \sum_{s=1}^{S-2}(\mathbf{W}\tilde{\mathbf{S}}_s^T)\Delta^2\mathbf{Y}_{t-s} + \mathbf{\Sigma}_t \qquad (13)$$

where: $\mathbf{\Gamma} = \mathbf{W}\tilde{\tilde{\mathbf{S}}}^T$, $\mathbf{\Psi}_s = \mathbf{W}\tilde{\mathbf{S}}_s^T$, $s = 1,\dots, S-2$,

but representation (13) has not yet been considered in the literature.

This paper assumes that co-autocorrelation is of order (1,1), which takes place when deviations from the short-run dependencies are AR(0), which is white noise, and first differences ($s = 1,\dots, S-1$) are stationary AR(1). In general, they are stationary AR(p) ($p \geq 2$). Co-serial correlation (again, by analogy to cointegration) may include all cases where deviations from the short-run dependencies are stationary AR(v) ($0 \leq v < p$).[25]

[25] **A. Hecq**, **F. Palm**, **J.-P. Urbain**, *Common Cyclical Features Analysis…*, pp. 117–141.

3. The role of full and reduced ranks in economics

Table 2 summarizes the role of common stochastic features in DGP and VAR models. Random walk is the “purest,” although a somewhat theoretical I(1) case. Most economic processes have long but not absolute memory, which means that most economic variables are generated by “near-random walk” processes.[26]

TABLE 2: *Full and reduced ranks in VAR models*

Matrices ranks	System features
1	2
Full rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables generated by purely random processes; a completely stagnant system
Full rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_s$	DGP – generating variables may contain stationary AR; variables are unrelated even in the short run
Full rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_s$	DGP – generating variables may contain stationary AR, but co-autocorrelation means that they do not translate into deviations from relationships between variables
Full rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1), and its increments are generated by purely random processes; there are only short-run relationships between variables
Full rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1), and there are only short-run relationships between variables; the DGP – generating variables contain stationary AR, which does not translate into deviations from relationships between variables due to co-autocorrelation
Full rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1), and there are no even short-run relationships between variables; the DGP – generating variables contain stationary AR, which is not annihilated due to the short-run dependencies absence
Full rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Reduced, non-zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1), and its increments are purely random; there are long-run equilibria between variables, from which deviations are whitened

[26] **A. Banerjee et al.**, *Co-integration, Error Correction and the Econometric Analysis of Non-stationary Data*, Oxford University Press, Oxford 1993.

1	2
Full rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Reduced, non-zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1), so its increments are purely random; there are long-run equilibria between variables, from which deviations are whitened, although the DGP – generating increments in the system's variables may contain stationary AR
Full rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Reduced, non-zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1), and its increments are purely random; there are long-run equilibria between variables, and deviations from these equilibria are stationary AR; no short-run relationships
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1) and I(2); there are medium- and short-run relationships between the system's variables, and the deviations from these relationships are purely random
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1) and I(2); there are medium- and short-run relationships between the system's variables, the deviations of which are purely random; the DGP – generating increments in the system's variables contain stationary AR in the error term
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(1) and I(2); there are medium-run relationships between the system's variables, and deviations of these relationships are stationary AR; absence of short-run dependencies
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Reduced, non-zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1) and I(2); there are medium- and short-run relationships between the system's variables, the deviations from these relationships are purely random
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Reduced, non-zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1) and I(2); there are medium- and short-run relationships between the system's variables, and deviations from these relationships are purely random; the DGP – generating increments in the system's variables contain stationary AR
Reduced, non-zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Reduced, non-zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_s$	variables are generated by cointegrated I(1) and I(2); there are medium-run relationships between the system's variables, and deviations from these relationships are stationary AR; absence of short-run dependencies
Zero rank of $\mathbf{A}_\perp^T \mathbf{\Gamma} \mathbf{B}_\perp$ Zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_s$	variables are generated by non-cointegrated I(2); there are short-run relationships between the system's variables, and deviations from these relationships are purely random; no medium-run dependencies

TABLE 2 (cont.)

1	2
Zero rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_{s}$	variables are generated by non-cointegrated I(2); there are short-run relationships between the system's variables, and deviations from these relationships are purely random; the DGP – generating increments in the system's variables contain stationary AR; no medium-run dependencies
Zero rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_{s}$	variables are generated by non-cointegrated I(2); as the system does not contain any long-, short- or medium-run dependencies, it makes no economic sense
Zero rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Reduced, non-zero rank of $\mathbf{\Pi}$ Full rank of $\mathbf{\Gamma}_{s}$	variables are generated by cointegrated I(2); there are short-run relationships between the system's variables, and deviations from these relationships are purely random; no medium-run dependencies
Zero rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Reduced, non-zero rank of $\mathbf{\Pi}$ Reduced, non-zero rank of $\mathbf{\Gamma}_{s}$	variables are generated by cointegrated I(2); there are short-run relationships between the system's variables, and deviations from these relationships are purely random; the DGP – generating increments in the system's variables contain stationary AR; no medium-run dependencies
Zero rank of $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ Reduced, non-zero rank of $\mathbf{\Pi}$ Zero rank of $\mathbf{\Gamma}_{s}$	variables are generated by cointegrated I(2); there are short-run relationships between the system's variables, and deviations from these relationships are stationary AR; no medium-run dependencies

S o u r c e: created by the author.

The $\mathbf{\Gamma}_{s}$ rank is largely independent of the rank of other matrices and is dependent on whether or not co-autocorrelation is present. This co-autocorrelation may not affect the cointegration of processes. The $\mathbf{\Pi}$ full rank excludes the $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ reduced rank problem. Additionally, because both $\mathbf{A}$ and $\mathbf{B}$ are $M * M$, then by definition $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp} = \mathbf{0}$. Summing up, a double or even triple reduced rank problem may occur. $\mathbf{\Pi}$ is associated with long-run CI(2,2), CI(2,1), and CI(1,1), the last of which may be present in both an I(1) system and, less frequently, in an I(2) system. $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ is assigned to the medium-run relationships. This CI(1,1) type is closer to stochastic co-cyclicality than to classic long-run cointegration. $\mathbf{\Gamma}_{s}$ are responsible for the short-run relationships measure.

To ensure the transparency of Table 2, the $\mathbf{\Gamma}_{s}$ zero ranks were associated with stationary but serially-correlated deviations from the short-run relationships.

However, other types of not purely random stationary deviations, such as heteroscedasticity or ARCH, are also possible. To keep Table 2 simple, the reduced but nonzero $\mathbf{\Pi}$ rank was not further decomposed in the I(2) domain (hence, with an $\mathbf{A}_{\perp}^{T}\mathbf{\Gamma}\mathbf{B}_{\perp}$ reduced rank). If the $\mathbf{\Pi}$ rank is R_0 (the number of CI(2,2)), stocks achieve equilibrium as soon as the flows accumulating to these stocks reach equilibrium. If the $\mathbf{\Pi}$ rank is R_1 (the number of CI(2,1)), stocks achieve equilibrium later than the flows accumulating to these stocks (the former only in the long run, and the latter even in the medium run). CI(2,1) seems to better describe adjustment in the economy.

4. Achieving gradual equilibrium

The polynomial cointegration mechanism can be described as follows (cf. Figure 1). In the medium run:

1) CI(2,2) equilibrium is achieved between some stocks only and continues in the long run. It is associated with R_0 linearly independent dependencies directions.
2) CI(2,1) equilibrium is achieved between flows (but not yet between stocks). It is associated with R_1 linearly independent dependencies directions.
3) CI(1,1) “equilibrium” $\mathbf{B}_{1\perp}^{T}\Delta\mathbf{Y}_{t-1}$ between stock increments is achieved, but this equilibrium is not permanent. This medium-run cointegration can be treated more as eliminating stochastic co-cycles rather than stochastic trends. It is associated with P_1 linearly independent dependencies directions.

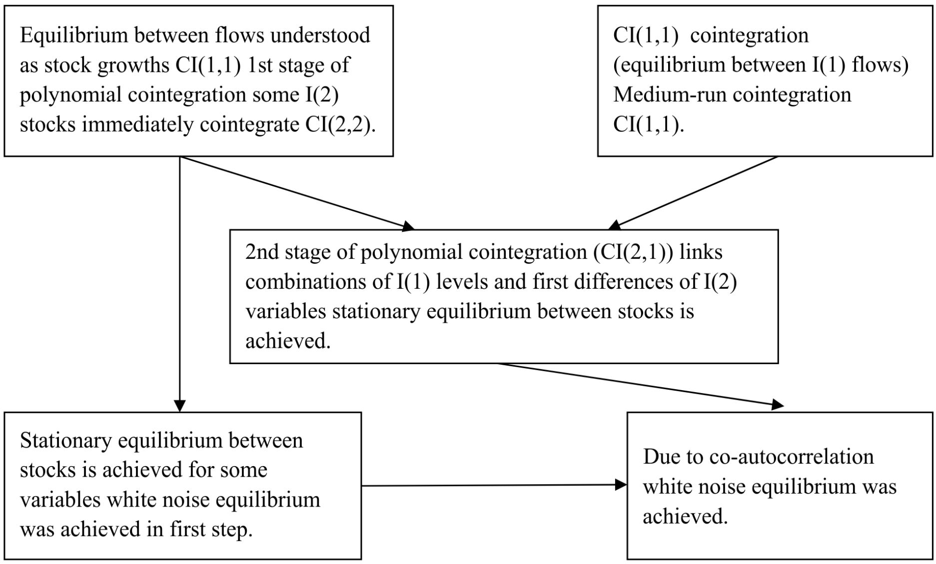
In the long run, the second step of polynomial cointegration involves eliminating those deviations from the equilibrium that were not eliminated in the medium run. These are:

a) R_1 deviations between I(2) stock levels that are still nonstationary (I(1)),
b) P_2 dependencies directions that cannot be interpreted as cointegrating ($\mathbf{B}_{2\perp}^{T}\Delta\mathbf{Y}_{t-1}$), i.e., stock increments (flows) that are not yet stationary but still I(1).

I(1) flows mentioned in point b) are not mutually cointegrated. In contrast to $\mathbf{B}_{1}^{T}\mathbf{Y}_{t-1}$, the $\mathbf{B}_{1\perp}^{T}\Delta\mathbf{Y}_{t-1}$ relationship clearly suggests that the respective variable levels are not cointegrated. However, the flows mentioned above can cointegrate in the CI(1,1) dependencies with deviations from the $\mathbf{B}_{1}^{T}\mathbf{Y}_{t-1}$ stocks relationships, which are obviously I(1). These deviations catalyze stock equilibrium in the system.

The last question is distinguishing between unlimited shocks in the $M-R$-dimensional non-cointegrated shocks space and mutually cointegrated shocks, which is R. The R_0 are shocks caused by either flows or stocks generated by I(1) processes, and the "full" equilibrium is achieved very quickly. The R_1 stock shocks cointegrate more slowly. First, flows adjustment is achieved, followed later by stock equilibrium. The latter case is known as classic polynomial cointegration.[27]

FIGURE 1: *Equilibrium achievement mechanism in model I(2)*



S o u r c e: created by the author.

The co-autocorrelation can be treated as the complementary stage to achieve stationary and whitened deviations from equilibrium.

5. Conclusions

The study demonstrated the purposefulness of a comprehensive analysis of co-features that occur in DGP–generating economic variables. Research has mostly focused on long-run shifts and less frequently on medium-run shifts, polynomial cointegration, or short-run relationships. Short-run relationships are usually

27 **S. Johansen**, *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press, Oxford 1995; **N. Haldrup**, *An Econometric Analysis of I(2) Variables*, Journal of Economic Surveys 1998/12, pp. 595–650.

considered separately from adjustment mechanisms that drive the system toward long-run equilibrium. The paper sought to determine how these common stochastic behaviors, particularly shifts that cause white noise deviations from the causal relationships that occur in the economy, are linked. The findings go beyond statistical importance. Super-consistency (or in I(2) domain super-super consistency) are important estimator features, but in the short-term analysis, estimator efficiency (ensured only by a correctly conducted analysis of the problems discussed in the paper) is more important. The benefits of the additional interpretative possibilities presented in the article are as important. Defining the mechanisms behind adjustments to the long- and medium-run equilibrium as a short-run reactions function allowed us to look at adjustment processes from a different angle and, importantly, capture their dynamics.

As regards short-run relationships, the actual space dimension of adaptive reactions that represent their special form can be (and usually is) different from the cointegration space dimension. The pattern is similar to the polynomial cointegration from the I(2) domain. Some stationary cointegration relationships are “whitened” to the extent that the deviations from them immediately become white noise. However, some adjustments combine mean-reversion with long-run equilibrium and short-run quasi-equilibrium. When the I(2) processes are present, the medium-run relationships cannot be ignored in the analysis.

Classical polynomial cointegration is a stock categories dependence with flow categories (more precisely, first increments in stock categories with “zero” increments in flow variables), but is a strictly CI(1,1) flow cointegration, which occurs in the medium horizon and becomes permanent in the long run. Therefore, in the polynomial cointegration relationship (the flows equilibrium persists to the stocks equilibrium). For dependency, the opposite situation occurs: only increases in stock categories (flows) cointegrate. The flows cointegration does not always lead to the relatively faster cointegration of stock categories (eliminating the I(1) stochastic trends still residing in them).

Because of editorial restrictions and the lack of appropriate software, many key issues could not be presented in-depth, so they were only outlined. They include the deterministic co-features, the seasonal cointegration full analysis, and the extension of co-autocorrelation and co-MA to higher degrees of AR or MA processes.

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WSPÓLNE CZYNNIKI STOCHASTYCZNE I ICH INTERPRETACJA EKONOMICZNA

Abstrakt

Przedmiot badań: Analiza kointegracyjna jest znana w literaturze od blisko 40 lat. Nieco mniej miejsca poświęca się innymi wspólnym czynnikom wytrącającym kategorie ekonomiczne ze stanu równowagi. W szczególności interesujące jest spojrzenie, w jakim stopniu wspomniane badania są względem siebie alternatywne, a w jakim komplementarne. Jakie warunki muszą być spełnione, aby podjęcie odpowiedniej analizy (kointegracyjnej, współcykliczności, współautoskorelowania czy innych, rzadziej stosowanych) było celowe.
Cel badań: W oczywisty sposób wybór rodzaju analizy wspólnych czynników (niekoniecznie) dominujących czy wynikającej z tego analizy współprzesunięć zależy od wyboru horyzontu analizy (długo-, średnio- czy krótkookresowej). Z drugiej strony rzetelne badanie nie powinno *a priori* pomijać żadnej z tych perspektyw. Starano się dowieść, że kluczową rolę odgrywają tu zredukowane rzędy najważniejszych macierzy, występujących w odpowiednich reprezentacjach VAR lub ich izomorficznych reprezentacjach. Innym celem badawczym było wykazanie, że wspomniane analizy współprzesunięć stochastycznych są w dużej mierze komplementarne względem siebie.
Metoda badawcza: Wybór metody badawczej wynikał z postawionej tezy. Wielowymiarowa ekonometria dynamiczna oparta na modelach VAR pozwoliła dostarczyć narzędzi służących porównaniu różnych metod analizy wspólnych czynników.
Wyniki: Rozpatrzone i zinterpretowane ekonomicznie zostały możliwe kombinacje pełnych i zredukowanych rzędów macierzy kointegrującej oraz macierzy związków średnio- i długookresowych. Ukazane zostały powiązania pomiędzy tymi macierzami. Rozrysowany został iteracyjny mechanizm powrotu systemu do równowagi. Potwierdzono, że rozważane analizy wspólnych czynników dominujących są w dużej mierze uzupełniające względem siebie, choć w znacznym stopniu wynika to z ograniczenia się (ze względu na przyjęte limity objętości) do dziedziny czasu oraz czynników stochastycznych. Rozszerzenie analizy o np. kointegrację sezonową czy współtrendowość deterministyczną z pewnością pozwoliłoby pokazać elementy substytucyjne. Przykładowo, analiza kointegracyjna w relatywnie ograniczonym horyzoncie czasowym może być alternatywą współtrendowości (trend stochastyczny wygasa dopiero w bardzo długiej perspektywie), również analiza uwzględniająca proces o wyższym stopniu zintegrowania mogłaby być alternatywą kointegracji sezonowej.

Słowa kluczowe: kointegracja, współautoskorelowanie, równowaga i mechanizmy dostosowawcze, szoki.